Counting and Measuring

Laurie Endicott Thomas, MA, ELS / Madison, NJ

To be a good medical writer, you need to know something about mathematics. Mathematics is the art of number, and numbers originated from words that were coined for the purpose of counting. However, some things can be counted, and some things cannot.

Count and Noncount Nouns

Discrete (as opposed to discreet!) means separate and distinct from other things. Objects that are discrete can be counted. For example, you might count the number of apples in a basket, but you can never count the number of gasoline in a tank. For this reason, *apple* is a count noun, but *gasoline* is a noncount noun. So you can ask, "How many apples are in the basket?" but it would be ungrammatical to ask, "How many gasolines are left in the tank?" Instead, you might ask, "How *much* gasoline is left in the tank?"

Many is used with count nouns; *much* is used with noncount nouns. Some quantifiers (eg, *all*, *any*, *enough*, *most*, *plenty of*, *some*, and *no*) can be used with count or noncount nouns. However, there are some quantifiers that are used only with count nouns (eg, *every*, *many*, *a few*) and others that are used only with noncount nouns (eg, *much*, *less*, *a little*).

If we want to talk about more than 1 of something in English, we use the plural form of the noun. The plural form is usually made by adding *s* or *es* to the end of the noun. However, there are many exceptions (see Table on next page). These include some words of Anglo-Saxon origin, such as *child/children*, or *woman/women*, *ox/oxen*, *goose/geese*. Note that many of the animal nouns that came from Anglo-Saxon are the same in singular and plural: *fish*, *sheep*, *moose*. Many words of Greek or Latin origin that are important in medicine have irregular plural forms: *bacterium/bacteria*, *corpus/corpora*, *genus/genera*, *medium/media*, *species/species*, *stigma/stigmata*.

For some nouns with irregular plurals and some noncount nouns, a regular plural form has become commonplace or is used in specific circumstances. For example, *water* is a noncount noun. However, the word *waters* is used to refer to a watery geographical area (eg, the navigable waters of the United States) or in some poetic contexts. Amniotic fluid, which surrounds the fetus in the womb, is also sometimes called waters.

Collective Nouns

A collective noun is a noun that refers to a group (set) of persons or things. For example, a swarm refers to a group of insects, and a choir refers to a group of singers. This raises problems of agreement with pronouns and verbs. Should you refer to the collective as "it" or "them"? Should you use the singular or plural form of the verb to refer to the collective's actions? The answer depends on whether the individual members or the group is being emphasized:

- The emergency department staff *are* trained in the latest resuscitation techniques (emphasizing individuals).
- The hospital's emergency department staff *is* the best in the city (emphasizing the group).

Note that British people are more likely than Americans to use plural pronouns and verbs for collectives, such as businesses:

- Bloomingdale's is having a sale on swimsuits (United States).
- Fenwick are having a sale on swimming costumes (Britain).

Units of Measure

Many things that cannot be counted can nevertheless be measured. To measure them, we need to find some unit of measure. For example, we can say "1 liter of water" or "2 bushels of wheat." Please notice the grammatical structure: the number

Table. Irregular English Plurals

Singular	Plural
addendum	addenda, also addendums
aircraft	aircraft
alumna	alumnae
alumnus	alumni
analysis	analyses
antenna	antennae, also antennas
antithesis	antitheses
apex	apices, also apexes
appendix	appendices, also appendixes
axis	axes
bacillus	bacilli
bacterium	bacteria
basis	bases
beau	beaux, also beaus
bison	bison
bureau	bureaus, also bureau
cactus	cacti, also cactus or cactuses
château	châteaux, also châteaus
child	children
codex	codices, also codexes
concerto	concerti, also concertos
corpus	corpora
crisis	crises
criterion	criteria, also criterions
curriculum	curricula, also curriculums
datum	data
deer	deer
diagnosis	diagnoses
die	dice, also dies
dwarf	dwarves, also dwarfs
ellipsis	ellipses
erratum	errata
faux pas	faux pas
fez	fezzes, also fezes
fish	fish, also fishes[a]
focus	foci, also focuses
foot	feet, sometimes foot
formula	formulae, also formulas
fungus	fungi, also funguses
genus	genera, also genuses
goose	geese
graffito	graffiti
grouse	grouse, also grouses
half	halves
hoof	hooves, also hoofs
hypothesis	hypotheses
index	indices, also indexes
lacuna	lacunae
larva	larvae
leaf	leaves
libretto	libretti, also librettos
loaf	loaves

Singular	Plural
locus	loci
louse	lice
man	men
matrix	matrices, also matrixes
medium	media, also mediums
memorandum	memoranda, also memorandums
minutia	minutiae
moose	moose
mouse	mice
nebula	nebulae, also nebulas
nucleus	nuclei
oasis	oases
octopus	octopuses or octopodes
offspring	offspring
opus	opera
ovum	ova
ox	oxen, also ox
parenthesis	parentheses
phenomenon	phenomena
phylum	phyla
quiz	quizzes
radius	radii
referendum	referenda, also referendums
salmon	salmon
scarf	scarves
schema	schemata, also schemas
self	selves
series	series
sheep	sheep
shrimp	shrimp, also shrimps[a]
species	species
stigma	stigmata
stimulus	stimuli
stratum	strata
swine	swine
syllabus	syllabi, also syllabuses
symposium	symposia, also symposiums
synopsis	synopses
tableau	tableaux, also tableaus
thesis	theses
thief	thieves
tooth	teeth
trout	trout, also trouts[a]
tuna	tuna, also tunas[a]
vertebra	vertebrae, also vertebras
vertex	vertices, also vertexes
vita	vitae
vortex	vortices, also vortexes
wharf	wharves, also wharfs
wife	wives
wolf	wolves
woman	women

[a] *The former is typically used to refer to more than 1 individual of the same species, and the latter is typically used to refer to more than 1 species.*

is an adjective modifying the unit of measure. The unit of measure is a noun. The material being measured is now the object of a prepositional phrase ("*of* water"). If you are talking about some noncount noun that is being measured in this way, it will be treated as if it were a singular: 20 kilometers is (not are) a long walk.

A unit of measure is arbitrarily defined; thus, the number associated with a measurement is meaningless unless the unit of measure has been defined, so you must carefully specify units of measure. Units of measure typically relate to some natural phenomenon. For example, the inch was originally based on the width of a man's thumb, and the foot was based on the length of a man's foot. The main basis of the International System of Units (SI, for Système International [d'unités]) is the meter. The SI grew out of the metric system that was developed in Revolutionary France. The meter was supposedly based on 1/10,000 of the distance from the North Pole to the Equator. Other units of measure in the SI were derived from the meter. A centimeter is 1/100th of a meter. A liter is 1,000 cubic centimeters, and a kilogram originally represented the mass of a liter of water. A newton is the amount of force to make a 1-kg object accelerate 1 meter per second per second. Thus, measurements of force involve units of time as well as units of mass and distance.

The SI also includes many units that are important in physics and chemistry. A coulomb (C) is a measure of electrical charge, and an ampere (A) is a measure of electrical current. A candela (cd) is a measure of luminous intensity. A mole (mol) is a measure of the amount of a substance. A mole is defined as $6.02214076 \times 10^{23}$ particles (eg, atoms or molecules). The number of particles in a mole is called Avogadro's number.

Our measurements of time were originally derived from the duration of a day. Each day is divided into 24 hours, and each hour into 60 minutes, each minute into 60 seconds. The 60-minute hour and 60-second minute are legacies of the ancient Mesopotamians, who used the number 60 as the basis of their number system. (The number 60 is the smallest number that can be divided evenly by every whole number from 1 to 6.) The ancients also divided a circle into 360°—partly because 360 can be evenly divided by so many different numbers and partly because 360 is close to the number of days in the year. (The lunar calendar has 355 days, and the solar calendar has 365 days). Thus, the sun would advance roughly 1° along the ecliptic (its circular path relative to the background of stars) every day.

Whole and Real Numbers

When we count objects, the result will be an integer. But when we measure the amount of something, as opposed to counting the number of items, the result would theoretically be a real number, along with a unit of measure. A real number is a number that can be expressed as some point along a number line. To express a measurement, however, we will end up using a rational number, as we will report only a limited number of digits after the decimal point. A rational number is one that can be expressed as a quotient or fraction of 2 integers. Its decimal expansion, if it has one, will either terminate or end up repeating itself endlessly. For example, ¾ = 0.75 and ⅓ = 0.33333…. (sometimes written $0.\overline{3}$, with the overbar representing the repeating decimal expansion). In contrast, irrational numbers have a decimal expansion that continues forever without repeating. Examples include π (the ratio of the circumference of a circle to its diameter), *e* (Euler's number, which is useful for calculating compound interest), and the square root of 2.

Using different units (eg, miles vs kilometers) will yield a different number, so you have to include the units with the number. Which unit of measure should you use? In scientific writing, you should use the SI (meters, kilometers, kilograms, etc). But if you are writing for consumers in the United States, you should probably use the units that are familiar to consumers (feet and inches, miles, pounds and ounces, etc).

Accuracy, Precision, and Uncertainty

Accuracy refers to how well a measurement agrees with the truth. In contrast, precision refers to the agreement among repeated measurements (made under the same conditions). Thus, a measurement that is accurate may be imprecise, and a measurement that is precise may be inaccurate. When choosing between methods of measurement, you often have to make a tradeoff between precision and accuracy. For example, a digital clock displays a precise, rational number, but that reading does not represent the true time. In contrast, an analog clock expresses time as a real number that cannot be read precisely.

Both inaccuracy and imprecision contribute to uncertainty. All measurements, and all quantities calculated from measurements, will have some degree of uncertainty. The degree of uncertainty can be expressed in various ways. One is by showing only a limited number of significant digits. For example, a reported value of 3.5 implies that the actual value is probably somewhere between 3.45 and 3.55. In contrast, a reported value of 3.50 implies that the actual value is probably somewhere between 3.495 and 3.505—a much narrower range. You can also express the uncertainty in units of measure or as a percentage of the total value: 25.2 mL ± 0.05 mL can be expressed as 2.52 mL ± 0.2%.

Even when we are dealing with counts, such as the number of people who live in a city, we sometimes have to deal with uncertainty. As a result, we may have to settle for an approximate number, such as when we say that the population of New York City was 8.40 million in 2018. Nor should we report too many digits after a decimal point: we shouldn't report a value as 5.38761 when the precision of the value really only lets us say 5.4.

Fractions and Percentages

A fraction is made by division. The top number (numerator) is divided by the bottom number (denominator). A percentage is a fraction whose denominator is 100. Whenever you encounter a percentage or any other fraction, try to figure out what the numerator and denominator represent. For example, the forced expiratory volume in 1 second (FEV_1) is the amount of air that a patient can exhale in 1 second and is measured in liters. This value can then be divided by the full forced vital capacity (FVC), which is the total amount of air that the person can exhale after taking the biggest possible breath, to yield the Tiffeneau-Pinelli index (FEV_1/FVC), which is a unitless rational number. The FEV_1 and FVC can also be expressed as a percentage of the values that are predicted, given the patient's sex, age, height, and race.

When talking about values that are already expressed in percentages, be cautious about using percentages to express changes. For example, if a value increased from 10% to 20%, that's an increase of 10 percentage points, not 10% (it's a 100% increase; see Percentage Increase and Decrease).

Negative Numbers and Vectors

Addition is the arithmetic operation that originally represented adding objects to a collection. Its opposite is subtraction, which originally represented the removal of objects from a collection. If you have 5 apples in a basket, you cannot remove more than 5 apples from that basket. But if you have $100 in your checking account and write a check for $200, you will end up with a balance of –$100 in your account. You would have to deposit $100 in the account to bring the balance up to 0. Accountants sometimes use parentheses instead of a minus sign to indicate negative numbers.

Addition and subtraction are often represented by rightward or leftward movement, respectively, on a number line. Thus, addition and subtraction involve not just quantity but direction. In mathematics, a geometrical object that has a direction as well as a magnitude is called a vector. A line is one-dimensional, so there are only 2 directions. In contrast, a map is two-dimensional, which allows for an infinite number of directions. If I walk 1 block north, then 1 block west, then 1 block south, then 1 block east, I will have walked a distance of 4 blocks, but I will end up back where I started. Human beings can easily think in terms of 4 dimensions: the 3 dimensions of Euclidean geometry plus time. However, mathematicians often deal with problems that involve more than 4 dimensions. This allows them to develop a mathematical model of relationships among many variables at the same time.

Exponents and Logarithms

Exponentiation is when you multiply a base number (*b*) by itself n number of times (b^n). For example, $2^3 = 2 \times 2 \times 2 = 8$. The n is called an exponent, and we often say that *b* has been raised to the nth power. If n = 2, we say that the base is squared. If n = 3, we say that the base is cubed. Medical communicators often deal with powers of 10: $10 = 10^1$, $100 = 10^2$, $1{,}000 = 10^3$, $10{,}000 = 10^4$, and so on. However, any real number could serve as the base or the exponent.

The use of exponents can turn a multiplication problem into an addition problem: if 2 exponential expressions have the same base, you can multiply the 2 by adding their exponents: $10^2 \times 10^3 = 10^5$. To divide, you subtract the exponents: $10^5 \div 10^3 = 10^2$. Any nonzero number divided by itself is 1; therefore, $b^0 = 1$. You can also have negative exponents: $b^{-n} = 1/b^n$. You can also raise negative numbers to any power. Note, however, that if you raise a negative number to an even power (eg, –1 × –1), the product will be a positive number. If you raise it to an odd power, the result will be a negative number (eg, –1 × –1 × –1 = –1).

In medical writing, you will often see powers of 10, especially in scientific notation: $5.23 \times 10^5 = 523{,}000$, but $5.23 \times 10^{-5} = 0.0000523$. Sometimes E notation is used to express powers of 10. 5.23E5 means 5.23×10^5, and 5.23^{E-5} means 5.23×10^{-5}.

A logarithm is the inverse function of exponentiation. If $b^n = x$, then $n = \log_b(x)$. Because $2^3 = 8$, $\log_2(8) = 3$. Because exponents can be negative, you can have negative logarithms, which represent the inverse of a number. Because ½ is the inverse of 2, the $\log_2(½) = -1$. Likewise, the $\log_{10}(1/10) = -1$ (Figure on next page).

Some units of measure are based on a logarithmic scale. For example, pH is based on the negative of the base-10 logarithm of the activity of the H^+ ion (as measured in moles per liter).

$$pH = -\log_{10}(a_{H^+}) = \log_{10}\left(\frac{1}{a_{H^+}}\right)$$

A solution of pure water has hydrogen activity of 1×10^{-7} mol/L. The reciprocal of that is 1×10^7, or 10^7; $\log_{10}(10^7) = 7$. So the pH of pure water is 7. Water with a pH of 6 would have a hydrogen activity of 1×10^{-6} mol/L, which is 10 times as many hydrogen ions as in pure water!

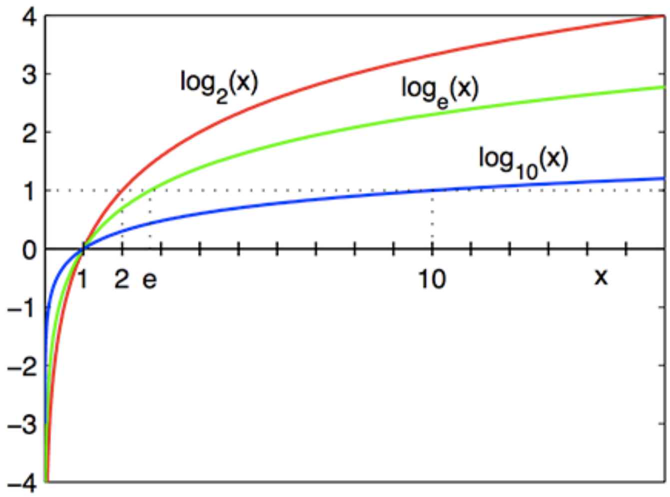

Figure. You can raise any real number (b) to any real power (y), even negative and fractional powers. The logarithmic function is the inverse function of exponentiation: If $b^y = x$, then $y = \log_b(x)$. The graph shows the value of $\log_b(x)$ for $x>0$ and some nonzero values of b. The value of $\log_0(x)$ is undefined; but for $b\neq0$, $\log_b(1)=0$ and $\log_b(b)=1$. Also, $\log_b(x) = -\log_b(1/x)$. Thus, the value of $\log_b(0)$ is undefined but approaches $-\infty$ as x approaches 0. The logarithm of a negative number is not a real number but involves a complex expression. *Graph courtesy of Richard F. Lyon via Wikimedia Commons.*

Base-10 logarithms are used so often that they are often just written as log(x). The natural logarithm, abbreviated ln(x), has Euler's number (e) as its base. Euler's number is an irrational number that is useful in many different areas of mathematics.

Medical writers should be aware that viral load is often expressed in base-10 logarithms. I once edited a news article that described a patient as having a viral load of 5 copies/mL. That value was dubious: a value that low had to be below the limit of detection of any available assay. When I looked at the source material, I found out that the patient's reported viral load was actually 5 $\mathbf{log_{10}}$ copies/mL, which meant 100,000 copies/mL. Big difference!

Stevens' Taxonomy of Measurement

Medical writers must be aware that numbers do not always represent counts, or some point along a number line, or a vector. To explain this problem, Stanley Smith Stevens explained that there are 4 types of measurement scale.[1]

- **Nominal—** A nominal scale is used when items or individuals are being assigned to groups that do not overlap. Such groups may be labeled with numbers: group 1, group 2, group 3. However, these numbers are simply being used as labels and do not express any sort of quantity.
- **Ordinal—** An ordinal scale is used when items or individuals are being ranked according to how they compare with each other in terms of some property. For example, the runners in a race will be ranked first, second, third, and so on, according to how fast they ran. However, these ordinal numbers merely show rank. They do not use units of measure, and they do not show absolute quantities or ratios. For example, the second-place finisher in a race was faster than the fourth-place finisher—but *not necessarily twice as fast*. The Wong-Baker FACES pain rating scale[2] is an ordinal scale. So are the Likert scales that are used in opinion research (eg, 1 = strongly disagree, 2 = disagree, 3 = neither agree nor disagree, 4 = agree, 5 = strongly agree).

Percentage Increase and Decrease

The formula for calculating percentage increase and decrease is:

Percentage Increase = [(Final Value – Starting Value)/ |Starting Value|] × 100

If you weigh 50 kg and gain 100 kg, that's a 200% increase in weight:

[(150 – 50)/50] × 100 = 200%

But if you then lose that 100 kg, that's only a 67% decrease in weight:

[(50-150)/150] × 100 = –67%

Fold Increase and Decrease

A fold is a ratio between 2 values. The formula for calculating fold increase is:

Fold change (for increases) = Final value/Starting value.

If you weigh 50 kg and gain 100 kg, then that's a 3-fold increase in weight:

(150/50) = 3

A fold decrease is calculated as follows:

Fold change (for decreases) = –(Starting value/ Final value).

If you weigh 150 kg and lose 100 kg, then that's a –3-fold change (3-fold decrease) in weight:

– (150/50) = –3

However, some people use the fold increase formula for calculating fold decreases. As a result, they would describe a change from 150 kg to 50 kg as a 0.33-fold decrease in weight. So if you see someone express a fold decrease, make sure you know what they really meant!

- **Interval—** An interval scale not only orders items or individuals according to some characteristic but also establishes equal intervals between the units of measurement. This allows you to do some mathematical operations, such as calculating averages. However, the 0 point may be meaningless. For this reason, the measurements *cannot be expressed in ratio*s. For example, in the Celsius or centigrade scale, 0° was set to represent the freezing point of water, whereas 100° was set to represent the boiling point of water. However, the Fahrenheit scale sets 0° at a different point and uses different intervals. Water that is at 40 °C (104 °F) is warmer than water at 20 °C (68 °F), but it is *not twice as warm*! So don't express such a change in temperature as a multiple or a percentage.
- **Ratio scale—** A ratio scale has a meaningful 0 point as well as equal intervals. This allows you to calculate ratios. For example, you can say that 1 thing weighs twice as much as another, or that something costs twice as much as something else.

Conclusion

So if you are being asked to report on the meaning of the outcome measures used in a study, you must present the numbers accurately and specify the units (if any). You must also *think about what those numbers really mean*! For example, it is an established convention that the 0 point on a psychological or educational measurement is arbitrary and may be meaningless.

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***Author contact:** www.nottrivialbook.com; lthomas521@verizon.net*

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